

УДК 37

## HOW TO ASSIGN GRADES TO TASKS SO AS TO MAXIMIZE STUDENT EFFORTS

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### **Abstract**

In some classes, students want to get a passing grade (e.g., C or B) by spending the smallest amount of effort. In such situations, it is reasonable for the instructor to assign the grades for different tasks in such a way that the resulting overall student's effort is the largest possible. In this paper, we show that to achieve this goal, we need to assign, to each task, the number of points proportional to the efforts needed for this task.

**Keywords:** *Grade assignment, maximizing student efforts*

### **1. Formulation of the problem**

**In some cases, students try to minimize their efforts.** In the ideal world, students should apply the maximal effort when studying for all their classes. In reality, students usually have a limited amount of time.

As a result, while they concentrate their efforts on their major classes, they limit their efforts in other classes to a necessary minimum -- usually, the minimum effort needed to get a passing grade in this class.

This phenomenon is especially frequent when students take requires classes outside their major discipline – e.g., when engineering students take required

humanity classes or when biology majors take math and/or computing classes which are not directly related to their discipline.

**How can instructors increase the students' efforts in these classes.** For classes in which students minimize their efforts, instructors try to maximize the student efforts -- to make sure that even with the current attitude, the students learn as much of the topic as possible. Since all these students care about is their overall grade for this class, the only thing that the instructor controls is which proportion of the grade goes for each task. How can we assign these grades so as to maximize the student efforts?

**Towards a precise formulation of the problem.** The overall grade for the classes is usually computed as a weighted average of grades for different tasks, i.e., in effect, as the sum of the partial grades given for each task. The ideal case -- usually described by  $I = 100$  points -- corresponds to the case when the student gets the maximum possible number of points for each of the tasks.

Let  $n$  denote the total number of tasks, and let  $m(i)$  denote the maximum number of points that a student can get for each task. Then, we have  $I = m(1) + \dots + m(n)$ . Let  $e(i)$  denote the amount of effort (e.g., measured by the time of intensive study) that a student needs to get the maximum number of point  $m(i)$  in the  $i$ -th task, and let  $E = e(1) + \dots + e(n)$  denote the overall effort needed to get a perfect grade  $m(i)$  on all the tasks -- and thus, the perfect grade  $I$  for the class.

As we have mentioned, the students do not always apply the maximum effort in studying. Let  $a(i)$  be the actual effort that the student applies to the  $i$ -th task -- e.g., into studying the  $i$ -th part of the material. (A student may be studying more than needed, but we only count the time that the student studies for the corresponding task.) Since the effort  $e(i)$  already provides a perfect mastery of the  $i$ -th task, we assume that

$$a(i) \leq e(i).$$

In the first approximation, it is reasonable to assume that the number of points gained by the student is proportional to the student's effort. If the student applies the maximal effort  $e(i)$ , this student will get  $m(i)$  points. Thus, in general, for each effort  $a(i)$ , the resulting number of points  $g(i)$  is equal to

$$g(i) = a(i) * (m(i) / e(i)).$$

The student wants to minimize the overall effort  $a(1) + \dots + a(n)$  under the constraint that the overall number of points is greater than or equal to the passing value  $g(0)$ :

$$a(1) * (m(1) / e(1)) + \dots + a(n) * (m(n) / e(n)) \geq g(0).$$

Thus, we arrive at the following precise formalization of the problem.

**Precise formulation of the problem.** Let us assume that we are given values  $l$ ,  $e(1)$ , ...,  $e(n)$ , and  $g(0)$ . For each tuple  $m = (m(1), \dots, m(n))$  for which  $m(1) + \dots + m(n) = l$ , let  $E(m)$  denote the value  $a(1) + \dots + a(n)$  corresponding to the solution to the following constraint optimization problem:

Minimize  $a(1) + \dots + a(n)$  under the constraints

$$0 \leq a(i) \leq e(i)$$

and

$$a(1) * (m(1) / e(1)) + \dots + a(n) * (m(n) / e(n)) \leq g(0).$$

Our goal is to select a tuple  $m$  for which the corresponding overall effort  $E(m)$  is the largest possible:

Maximize  $E(m)$ .

*Comment.* This problem is an example of *bilevel* optimization problems, in which on the top level, we select the parameters of the objective functions so that the solution to the resulting low-level optimization problem will optimize an appropriate high-level objective function; see, e.g., [1].

In our case, the low level optimization is performed by a student, who is trying to minimize his/her efforts under the constraint that his overall number of points is at least  $g(0)$ . The grades  $m(i)$  for each task are parameters in this student's optimization problem. The instructor -- top-level optimizer -- would like to select these parameters in such a way that the resulting overall student's effort is as large as possible.

## 2. Solution to the problem

**Description of the solution.** As we show in this section, the optimal solution is to assign grades  $m(i)$  proportional to the effort, i.e., to have

$$m(i) = (l / E) * e(i).$$

**Proof.** For the above assignment, we have  $m(i) / e(i) = l / E$ , so for all possible actual efforts  $a(i)$ , the resulting grade is equal to

$$(m(1) / e(1)) * a(1) + \dots + (m(n) / e(n)) * a(n) = (l / E) * (a(1) + \dots + a(n))$$

and is, thus, proportional to the overall effort  $A = a(1) + \dots + a(n)$ . The student wants to minimize the overall effort under the condition that this grade is at least  $g(0)$ . The corresponding constraint  $(I / E) * A \geq g(0)$  is

equivalent to  $A \geq g(0) * (E / I)$ . Thus, the smallest possible value  $E(m)$  of the overall effort  $A$  is equal to

$$E(m) = g(0) * (E / I).$$

Let us prove that for any other grade assignment  $m' \neq m$ , we have  $E(m') < E(m) = g(0) * (E / I)$ .

Indeed, the assignment  $m$  is characterized by the fact that for this assignment, the ratio  $m(i) / e(i)$  is constant.

Since  $m' \neq m$ , for this new assignment, the ratio  $m'(i) / e(i)$  is not constant, it takes at least two different values for some  $i$ .

If we had  $m'(i) / e(i) \leq I / E$  for all  $i$ , then, since  $m' \neq m$ , we should have  $m'(j) / e(j) < I / E$  for some  $j$ . In this case, we have  $m'(i) \leq e(i) * (I / E)$  for all  $i$  and  $m'(j) < e(j) * (I / E)$  for some  $j$ . By adding all these inequalities, we get  $m'(1) + \dots + m'(n) < E * (I / E) = I$ , which contradicts the fact that for each grade assignment, we should have  $m'(1) + \dots + m'(n) = I$ . Thus, this case is impossible, and we have at least one index for which  $m'(i) / e(i) > I / E$ . Let us denote one of these indices by  $k$ , then  $m'(k) / e(k) > I / E$ , and  $m'(k) > e(k) * (I / E)$ . If we subtract this inequality from the equality  $I = E * (I / E)$ , then we get

$$I - m'(k) < (E - e(k)) * (I / E),$$

hence  $(I - m'(k)) / (E - e(k)) < I / E$ . From this inequality and the inequality  $m'(k) / e(k) > I / E$ , we conclude

that  $(I - m'(k)) / (E - e(k)) < m'(k) / e(k)$ . Taking the inverse of both sides, we conclude that:

$$e(k) / m'(k) < (E - e(k)) / (I - m'(k)),$$

thus

$$e(k) < m'(k) * ((E - e(k)) / (I - m'(k))).$$

For each  $a > 0$  and  $b > 0$ , let the student spend a little bit more effort on the  $k$ -th task than in the proportional assignment, i.e., take  $a(k) = ((g(0) / I) + a) * e(k)$ , while for other tasks, the student will spend a little less effort:  $a(i) = ((g(0) / I) - b) * e(i)$ . Under the grade assignment  $m'$ , the student's grade  $g$  will be equal to

$$g = ((g(0) / I) + a) * m'(k) + ((g(0) / I) - b) * m'(1) + \dots + ((g(0) / I) - b) * m'(k - 1) +$$


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$$((g(0) / l) - b) * m'(k + 1) + \dots + ((g(0) / l) - b) * m'(n)$$

Opening parentheses, combining terms proportional to  $g(0) / l$ , and taking into account that

$$m'(1) + \dots + m'(k-1) + m'(k) + m'(k+1) + \dots + m'(n) = l$$

and thus

$$m'(1) + \dots + m'(k-1) + m'(k+1) + \dots + m'(n) = l - m'(k),$$

we conclude that

$$g = (g(0) / l) * l + a * m'(k) - b * (l - m'(k)) = g(0) + a * m'(k) - b * (l - m'(k)).$$

We can get  $g = g(0)$  if we select  $b$  in such a way that  $a * m'(k) - b * (l - m'(k)) = 0$ . Then,

$$b = m'(k) / (l - m'(k)).$$

For this selection of  $b$ , the overall student's effort is equal to

$$A = a(1) + \dots + a(n) = a(k) + (a(1) + \dots + a(k-1) + a(k+1) + \dots + a(n)) = \\ ((g(0) / l) + a) * e(k) + ((g(0) / l) - b) * (e(1) + \dots + e(k-1) + e(k+1) + \dots + e(n)).$$

Opening the parentheses, combining terms proportional to  $g(0) / l$ , and taking into account that

$$e(k) + (e(1) + \dots + e(k-1) + e(k+1) + \dots + e(n)) = E$$

and thus

$$e(1) + \dots + e(k-1) + e(k+1) + \dots + e(n) = l - e(k),$$

we conclude that

$$A = g(0) * (E / l) + a * e(k) - b * (E - e(k)).$$

The first term in the right-hand side is exactly  $E(m)$  for the above grade assignment  $m$ . Substituting the above expression for  $b$  into this formula, we conclude that

$$A = E(m) + a * (e(k) - m'(k) * ((E - e(k)) / (l - m'(k)))).$$

We have already proven that  $e(k) < m'(k) * ((E - e(k)) / (l - m'(k)))$ . Thus, we have  $A < E(m)$ . By definition,  $E(m')$  is the smallest possible effort that the student needs to spend to get  $g(0)$ , thus  $E(m') \leq A$  and hence,

$$E(m') < E(m).$$

The optimality of the grade assignment  $m$  is thus proven.

This work was supported in part by the US National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science) and HRD-1242122 (Cyber-ShARE Center of Excellence), and by the Mexico SEP-CONACYT grants CB-2013-01-221676 and FC-2016-01-1938. This work was partially done when Vyacheslav and Nataliya Kalashnikovs were visiting researchers at the University of Texas at El Paso.

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*Материал поступил в редакцию 15 июля 2019 года*

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