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ANATOLE FRANCE'S STATEMENT ON EDUCATION TRANSFORMED INTO A THEOREM

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Abstract

Education researchers often cite a statement from Anatole France: "An education isn't how much you have committed to memory, or even how much you know. It's being able to differentiate between what you know and what you don't." In this paper, we show how this statement can be transformed into an exact theorem.

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Education researchers often cite a statement from Anatole France: "An education isn't how much you have committed to memory, or even how much you know. It's being able to differentiate between what you know and what you don't." In this paper, we show how this statement can be transformed into an exact theorem.

Suppose that we have a formal theory T from which we can deduce different statements. In this case, for each statement S, having knowledge about S means that we can either deduce the statement S or deduce its negation ~S. The famous Goedel's theorem states that if a theory T is strong enough (e.g., if it contains arithmetic), then for some statements S, we cannot deduce neither S, nor ~S from this formal theory. For such statements, within this theory, we do not have knowledge.

It turns out -- see below -- that if we can tell, for each statement, whether we have knowledge about it about it or we don't, then, based only on this information, we can determine which statements are true and which are false in this theory. In other words, if we are able to differentiate between what we know and what we don't, then, based on this differentiation ability, we can reconstruct the full knowledge. It is natural to call this statement -- formalizing what Anatole France said – Anatole France's theorem.

Definition 1. *Let T be a formal theory, and let S be a statement in this theory. We say that in the theory T,* we have knowledge about S *if from the theory T, we can derive either the statement S or its negation ~S.*

Definition 2. *We say that a theory T is* sufficiently strong *if there exists a statement S about which we do not have knowledge in the theory T.*

Anatole France's Theorem. *Let T be a sufficiently strong theory. Let us assume that we have a method that, for each statement S, determines whether in the theory T, we have knowledge about S or not. Then, by using this method, we can determine, for each statement S about which we have knowledge, whether S or its negation ~S is derivable in the theory T.*

Proof. We assume that we have a method that, given a statement S, determines whether we have knowledge about this statement or not. Based on this method, we want to produce another method -- that, given a statement S about which we have knowledge, determines whether S or ~S are derived from the theory T.

This desired method can be described as follows. Since the theory T is sufficiently strong, there exists a statement U about which we have no knowledge. We can find such a statement if we consider all possible statements one by one and check, for each statement, whether we have knowledge about this statement or not; eventually, we will find such U about which we do not have knowledge.

Then, for each statement S about which we have knowledge, we consider an auxiliary statement S & U.

- If S is false in T, then S & U is also false and thus, in the theory T, we have knowledge about S & U.
- On the other hand, if S is true in T, then S & U is simply equivalent to U.

Thus, if we had knowledge about S & U, we would also have knowledge about U -- and we have selected U as a statement for which we do not have knowledge.

So, if we have knowledge about S, then S is false if and only if we have knowledge about the auxiliary statement S & U. Hence, to find out whether S or its negation ~S are derivable in the theory T, all we need to do is check whether, in this theory, we have knowledge about S & U. The statement is proven.

Comment. If a theory T is not sufficiently strong, then, in this theory, we have knowledge about every statement. Thus, the ability to differentiate when we have knowledge and when we don't is trivial -- we have knowledge about every statement. Thus, this ability does not provide us with any information that would help us decide which statements are true and which are false.

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