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## FROM MISTAKES, WE LEARN: TEACHERS' POSITIONAL FRAMING TOWARD ERRORS IN MATHEMATICAL CLASSROOMS

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### ***Abstract***

The productive status of errors is discussed in Mexico and the US mathematics education reforms. However, teachers' positionings toward mistakes may or may not converge with this productive status. For that reason, reflecting on teachers' positioning during the teaching and learning process is crucial (Stooksberry et al., 2009). This study examines teachers' positional framing during teacher and students' moment-to-moment interaction in the context of errors occurring in the classroom. Findings revealed two opposite error frames that teachers used to address errors in their classrooms. One of these frames provided student support for using errors as tools for their learning. On the contrary, the other provided an idea of student incapacity to cope with errors.

***Keywords:*** *disposition, positioning, errors*

"From mistakes, we learn" it is a well-known phrase, however, in the education context, it seems to be a contradiction. Indeed, it is in the school where mistakes are frowned upon and punished by teachers by using grades. From the student's perspective errors are perceived as sources of anxiety, shame, and stress. Fortunately, that situation is not experienced in all the mathematics classrooms; it is fair to say, that it has been gradually changing.

Using errors as learning opportunities have been pointed out as essential part of learning mathematics since they promote a deeper understanding and support students' reasoning abilities. Research addressing this new approach shows an important trend in the role of errors in teaching and learning processes as instruments that promote a deeper understanding and analysis of mathematical concepts (Borasi 1987,1994; Bray & Santagata 2014; Kramarski & Zoldan, 2008; Schleppenbach, Fle-

vares, Sims, & Perry, 2007; Tsovaltzi et al., 2010; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011).

Even though, learning from errors it is not just about addressing students' errors, but how students' errors are addressed by their teachers what might support students' learning from their mistakes. However, notwithstanding there is wide evidence about the benefits of learning mathematics from an error positive status perspective, there are teachers that still perceiving those as learning deficiencies. In other words, teacher's positioning when a student makes a mistake become decisive for taking advantage of errors. Students' learning from mistakes depends on teachers' reactions toward them (Gojak, 2013). Hence, it can be assumed that teachers' disposition and positioning toward error in the classroom is likely to influence students' attitudes towards learning from mistakes and, therefore, their ability to do so (Steuer & Dresel, 2011; Tulis, 2013).

Understanding teachers' disposition toward mistakes become essential for examining the difference between productive and non-productive dis/position in classroom when those become active by the type of frames that teachers enact during the specific moment that a student errs. Consequently, this study addressed secondary mathematics teachers' disposition toward their own mistakes and their students' mistakes in the context of the U. S. and Mexico border.

## **1. THEORETICAL AND CONCEPTUAL FRAMEWORKS**

The underpinnings of this study are the theoretical and contextual frameworks which were grounded in the disposition toward mathematics framework (Beyers, 2011) and Positional framing (Greeno, 2009). Beyers (2011) provides a framework to study students dis/positions toward mathematics that classifies mental processes from this tripartite approach that involves cognitive, affective, and conative elements. He states these three functions as what constitutes disposition toward mathematics. Disposition is understood as the core of the dis/position reciprocal relationship "dispositions are at the root of teachers' decisions to think and to act" (Schussler, 2006).

Regarding, theoretical frameworks addressing participants' positioning and how the other participants are positioned by them allows researchers to examine and analyze what people are doing on moments of action and interaction in a specific

context and dynamics nature (Harré, 1995; Harre & Slocum, 2003). Positioning allows scholars to examine teachers' kinds of participation according to what they say and do in their classrooms at the specific moment that mistakes emerged and are addressed or not by teachers (Wagner & Herbel-Eisenmann, 2009). Greeno offers some theoretical assumptions that allow researchers to understand, conceptualize, and frame concepts. He distinguishes two aspects of framing: epistemological and positional framing. Positional framing refers to the ways in which participants positioning themselves and the others when in the activity they are interacting in, and framing is being constructed in a particular context. In this respect, when teachers establish an act (disposition) privileging some positioning over others, they also establish who can do what.

## **2. METHODOLOGY**

This study was conducted by using an explanatory sequential mixed methods design which is also known as sequential triangulation or integration (Morse, 1991). This type of design is composed of two phases – quantitative and qualitative (Creswell et al. 2003). Using both methods within the same project ensure a deeper understanding of the problem. The explanatory sequential design is mainly used because the quantitative results are crucial for planning the qualitative phase. In other words, the second phase, the qualitative phase of the study was built on the quantitative phase, which in turn, become connected in every part of the study.

The quantitative phase analysis provided a general overview of teachers' disposition toward errors and the qualitative phase provided an in-depth and exhaustive understanding the relationship between teachers disposition and teachers positioning in the error's episodes during class. It is important to specify that this paper focuses on the qualitative results of the study.

In the second phase, a multi-case study approach was used, with the aim of distinguishing between productive and non-productive dispositions toward mistakes across three secondary mathematics teachers. The main data sources applied here were participant observations (Musante & DeWalt, 2010) and in-depth semi-structured face-to-face interviews with the three participants. Data collection during the observations was to follow teacher's positioning during the error episodes in their classrooms. Then, fields notes were centered on teacher-student conversations,

questions, commands, and statements in the moments when errors emerged. In the same way data analysis was performed by focusing on teacher's positioning and how they positioned their students at the moment of errors and how these positioning facilitated or inhibited using errors productively.

### **3. RESULTS**

Explaining the variations between teachers positional framing when errors emerged during class was the purpose of the qualitative data collection and analysis. About the interviews, it is important to emphasize that all teachers expressed not only to understand the positive role that errors play on student mathematical learning which has been introduced and stated by math-reforms, but a personal commitment to use their students errors as tools for learning. All the teachers provide enough information to established that they appreciate errors as tools for learning from a cognitive perspective. Although, during the error episodes that took place in the classroom moment-to-moment interaction some of them remained entangled and replicating errors as deficiencies of learning frames. Then, during class observations, what previously was expressed by teachers about errors, was not aligned with some of their teaching practices. Hence, I was focused on how teachers framed their own mistakes and their students' mistakes, with the aim of knowing how teachers' disposition was reflected when errors emerged.

The way that teachers framed errors and how they positioned themselves and their students was interpreted by contextualizing them according to errors as resources to learn and errors as learning deficiencies; these two opposite error mathematics education paradigms that have been led all around the world by mathematics teachers though all levels of education and have been well documented by researchers, as well. In this way, teacher error handling practices did not approach from a deficit perspective by underestimating the efforts that teachers made for accomplishing reform requirements and minimizing the challenges that teachers face in their attempt to transform their practice.

Consequently, teachers framing were examined in the light of the paradigm that stresses error importance from a remediation perspective, from error pattern diagnosis (e.g., Ayres, 2001; Brousseau, 2006; Del Puerto, Minnaard, & Seminara, 2006) and the paradigm that supports the idea of treating errors as tools that pro-

mote student mathematical deeper understanding and create learning opportunities (Borasi, 1987/1994; Booth, Lange, Koedinger, & Newton, 2013; Bray & Santagata 2014; Schleppenbach, Flevaras, Sims, & Perry, 2007; Tsovaltzi et al., 2010; Tulis, 2013; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011).

The error handling practices that were identified for each of the three teachers were used for generating a matrix of frames (Table 1.1). Those practices are displayed as bulleted items.

Table 1.1. Ways of framing errors

<b>WAYS IN WHICH TEACHER COMMUNICATE THEIR ACTIVE DISPOSITIONS TOWARD MISTAKES</b>	
<b>Errors as resources to learn Frame</b>	<b>Errors as deficiencies of learning Frame</b>
<p><i>Understanding and analyzing mistakes, develop a critical thinking built-in error. Ability of capitalizing errors.</i></p> <ul style="list-style-type: none"> <li>• Instructional strategies and activities involve error’s analysis</li> <li>• Errors are addressed as a planned activity</li> <li>• Teacher communicates and anticipates errors</li> <li>• Teacher differentiates between different types of mistakes</li> <li>• Systematic connection between error analysis and learning</li> </ul>	<p><i>Understanding errors as learning deficiency. Using errors for diagnosing or remediate learning problems.</i></p> <ul style="list-style-type: none"> <li>• Focusing on correctness as established activity</li> <li>• Personalizing mistakes by isolating them for the rest of the group</li> <li>• Teacher corrects errors by him/herself</li> <li>• Teacher avoids and prevents errors</li> <li>• Discussing solution errors to routine problems</li> <li>• Explicitly valorizing speed and correctness (Louie)</li> <li>• Systematic connection between error and bad grades.</li> </ul>
<p><i>Flexibility and openness toward mistakes creating an error-friendly belief.</i></p>	<p><i>Reluctance toward mistakes creating an error-discomforting belief</i></p>

<ul style="list-style-type: none"> <li>• Teacher discusses mistakes openly</li> <li>• Explicitly states errors usefulness on learning</li> <li>• Discussion of errors as part of the socio-mathematical norms</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher covers up mistakes</li> <li>• Set errors as the result of lack of ability and practice</li> <li>• Focusing discussions exclusively on answers (Louie)</li> </ul>
<b>WAYS IN WHICH STUDENTS ARE EXPECTED TO PARTICIPATE</b>	
<p><b>Teacher position their student as capable of coping with errors. Students as competent and qualified to handle their error analysis process by themselves</b></p> <p><b>Frame</b></p>	<p><b>Teacher position their students as not capable of coping with errors</b></p> <p><b>Frame</b></p>
<p><i>Student as capable of producing mathematical ideas from the analysis of their mistakes</i></p> <ul style="list-style-type: none"> <li>• Students are encouraged to seek and value alternative ways of the error analysis process</li> <li>• Students show initiative</li> <li>• Students are involved in the error correction process</li> </ul>	<p><i>Students as receivers of mathematical ideas in regard to the correction of their mistakes</i></p> <ul style="list-style-type: none"> <li>• Students passive wait for teacher and/or peer correction of their mistakes</li> <li>• Students see their mistakes as flaws of their ability to learn mathematics</li> </ul>
<p><i>Student as capable of succeeding again before having experienced failure or having made a mistake</i></p> <ul style="list-style-type: none"> <li>• Students rely on others to correct a mistake through discussion</li> <li>• Students being confident and seeing as valuable discussing and getting agreements when they are working with others.</li> </ul>	<p><i>Students as vulnerable participants or/and not capable before having experienced failure or have made a mistake</i></p> <ul style="list-style-type: none"> <li>• Students reluctance to communicate their mistakes</li> </ul>

This matrix encompasses the different ways that the participants positioned themselves and their students at the moment that errors arose by framing them as tools or as deficiencies for learning. Besides, how these teachers expected their students to participate by positioning them as capable to cope with their own and their peer's mistakes or as weak pieces of the learning context due to their mistakes, from which they should stay away.

Become evident that, despite teacher's understanding of mathematical reform that proposes a new status for errors in student math learning and their understanding and commitment to using errors productively in their classrooms which were expressed during their interviews, their attempts to apply teaching strategies that incorporate error analysis, at times evidenced a tension between the two opposite error paradigms. Furthermore, without consistent, deliberate attention to teacher framing, much of some of the participant's teaching practices had the unintentional and inadvertent effect of perpetuating correctness as a paramount (Louie, 2017). This case provided an example to the difficulties that teachers face when error positive status is emphasized at a reform level.

#### **4. CONCLUSION**

Two opposite error frames that teachers used to address errors in their classrooms were identified. One of these frames provided student a clear idea of using errors as tools for their learning. The other, instead, provided an idea of student learning deficiency, lack of knowledge resulting on erring. For using errors as learning tools, it should become crucial that teachers see all their students as capable to learn and not evaluate their mathematical ability according to the number of errors that they make or the speediness to solve a problem correctly. Mathematics teachers need to divest of their narrow understanding of mathematical ability that gives rise to a non-productive disposition toward mistakes. A narrow understanding of the mathematical ability that leads to making a difference between those students that make mistakes and those who are commonly correct. Those conceptions conduct teachers to avoid using errors as springboards for learning opportunities.

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