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MIXED METHODS STUDY OF MIDDLE SCHOOL MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN USA AND RUSSIA USING SEQUENTIAL NESTED DESIGN

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Abstract

The sequential nested mixed methods study focused on comparative analysis of middle school mathematics teachers' content knowledge in two countries. The study consisted of two stages: (1) quantitative study of teacher content knowledge; (2) qualitative study of teacher topic-specific content knowledge. The initial sample for the first stage included lower secondary mathematics teachers from the U.S. (grades 6–9, N=102) and Russia (grades 5–9, N=97). The Teacher Content Knowledge Survey (TCKS) was applied to assess teacher content knowledge based on the cognitive domains of Knowing, Applying, and Reasoning, as well as addressing the lower secondary mathematics topics of Number, Algebra, Geometry, Data and Chance. The second stage – an interpretive cross-case study – aimed at the examination of the U.S. and Russian teachers' topic-specific knowledge on the division of fractions. For the second stage, N=16 teachers (8 – from the U.S., and 8 – from Russia) were selected for the study using non-probability purposive sampling technique based on teachers' scores on the TCKS. Teachers were interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge. The study revealed that there are explicit similarities and differences in teachers' content knowledge as well as its cognitive types. The study results may inform the field on priorities placed on lower secondary mathematics teachers' knowledge in the USA and Russia. It also suggests close comparison and learning about issues related to teacher knowledge in both countries with a potential focus on re-examining practices in teacher preparation and professional development.

***Keywords**: cross-national comparison, teacher knowledge, topic-specific content knowledge, lower secondary school mathematics*

INTRODUCTION

Cross-national studies allow understanding of how teacher education is contextualized in selected countries which requires "a range of analytical methods that draw out conflicting views, contested areas, and shared order to create "a more balanced comparative perspective" in teacher preparation across countries (Kim beliefs" (LeTendre, 2002). In last decade, a number of cross-national studies on teacher education were focusing on unpacking "culturally contextualized and semantically decontextualized dimensions" in Ewha, Ham, Paine, 2011). Scholars have addressed characteristics such as teachers' perceptions of effective mathematics teaching (Cai, Wang, 2010), role of opportunity to learn in teacher preparation (Schmidt, Cogan, Houang, 2011), teacher education effectiveness (Blomeke, Suhl, Kaiser, 2011), teachers' epistemological beliefs on nature of mathematics (Felbrich, Kaiser, Schmotz, 2012), and other issues. A number of papers addressed these issues at the pre-service teacher preparation level (Tatto, Senk, 2011; Felbrich, Kaiser, Schmotz, 2012). However, few comparative studies focused on in-service teachers' content knowledge. Moreover, the field lacks research that provides an in-depth analysis of teacher knowledge at a topic-specific level. Therefore, this study attempted to examine the U.S. and Russian in-service teachers' content knowledge through the lens of topic-specific context – a division of fractions.

The motivation for the study is based on the 8th-grade mathematics portion of the TIMSS-2015 results (Mullis et al., 2016). We identified two countries ranked closely to each other: Russia – in the 6th position and the USA – in the 10th position. At the same time, a difference in the U.S. and Russian students' scores was revealing: the average score of Russian students in the content domain was 538 (SE=4.7) and of the U.S. students – 518 (SE=3.1), with Russian students gaining higher scores on Number (533 vs. 520), Algebra (558 vs. 525), and Geometry (536 vs. 500) whereas the U.S. students outscored Russian students in the domain of Data and Chance (522 vs. 507). Russian students also outperformed the U.S. students in each cognitive domain: Knowing (543 vs. 528), Applying (541 vs. 515), and Reasoning (528 vs. 514). These data triggered the following *research question*: to what extent is the U.S. and Russian middle school mathematics teachers' knowledge differ by content and cognitive domains? Considering the importance of teachers' topic-specific knowledge (Ball, 1990; Ma, 1999), the study also zoomed into the *question*: to what extent is the U.S. and Russian lower secondary mathematics teachers' content knowledge similar and/or different in the topic-specific context?

The paper includes several sections. First, we provide an extended literature review in the field of cross-national studies in teacher education and teacher knowledge. Then we discuss the methodology of the study which consists of the research design, participants, procedure, data collection, and analysis. Finally, we will present the results of the study followed by a discussion and conclusion.

CROSS-NATIONAL STUDIES ON MATHEMATICS TEACHER KNOWLEDGE

Conducting cross-national studies allow comparing, sharing, and learning about issues in an international context (Robitaille & Travers, 1992). Cross-national studies also help researchers understand in a more explicit way about their own context, teaching practice, knowledge, and get insights of better choices in constructing the teaching and learning process (Stigler and Perry, 1988). In this section, we will discuss recent studies in mathematics teacher education within the cross-national context. Studies vary in a scope addressing different issues including but are not limited to general aspects in teacher education, teacher knowledge, different types of teacher knowledge, connections between teacher knowledge and student performance, instrument development and adaptation, to name a few.

Analysis of a body of literature in cross-national research in mathematics teacher education and teacher knowledge demonstrates that few comparative studies focused on in-service teachers' content knowledge in general, and within topic-specific context – in particular. Addressing this deficiency, the proposed study attempts to closely examine the U.S. and Russian lower secondary school mathematics teachers' content knowledge through the lens of topic-specific context – the division of fractions.

The field of mathematics teacher education is expanding its knowledge-base in understanding the role of teacher characteristics in student learning and achievement. The major shift in the field had happened with Shulman's (1986) work on teacher knowledge that proposed an alternative approach to the educational production function perspective (e.g., Hanushek, 1981, Monk & Rice, 1994), which was concerned with examining proxies of teacher knowledge such as coursework/certification and its impact on student achievement (Charalambous & Pitta-Pantazi, 2016). Research on teacher knowledge initiated by the work of Shulman (1986) has focused on teacher knowledge as a major predictor of student learning and achievement. In the last decade, the field benefited from numerous studies (Hill, Shilling, & Ball, 2004; Hill, Ball, & Schilling, 2008; Rowland, Huckstep, & Thwaites, 2005; Davis & Simmt, 2006; Baumert et al., 2010) that substantially advanced the conceptualization of teacher knowledge.

Capitalizing on Shulman's (1986) work, scholars examined different categories of teacher knowledge. Content or subject-matter knowledge and pedagogical content knowledge are the most important categories of teacher knowledge. Bransford, Brown, and Cocking (2000) state that content knowledge requires "a deep foundation of factual knowledge, understanding of the facts and ideas in the context of a conceptual framework, and organization of the knowledge in ways that facilitate retrieval and application" (p. 16). Hill, Ball, and Schilling (2008) consider a special kind of teacher knowledge that combines content and pedagogical content knowledge – mathematical knowledge for teaching. It is knowledge "that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (p. 378).

Some scholars (e.g., Chapman, 2013; Izsak, Jacobson, & de Araujo, 2012) examined different facets of teacher knowledge without explicitly emphasizing its connection to student learning. Other scholars stressed the importance of the kind of knowledge a teacher possesses because it impacts his/her teaching (Steinberg, Haymore, and Marks, 1985). Another line of research (e.g., Hill, Rowan, & Ball, 2005; Baumert et al, 2010; Author, 2011) specifically targets the effects of different types of teachers' knowledge on student achievement. There is a need in the field for extending the latter line of research to the level of cross-national studies on teacher knowledge.

Recently, scholars have advanced the field by examining teacher knowledge in variety of domains including Number Sense (Ma, 1999; Izsac, 2008), Algebra (Bair & Rich, 2011; McCrory et al., 2012), Geometry and Measurement (Murphy, 2012; Nason, Chalmers, & Yeh, 2012), and Statistics (Groth & Bergner, 2006). However, the field lacks cross-national research that provides a comprehensive analysis of the various facets of teacher knowledge including content and cognitive domains as well as its granualization in the topic-specific context.

METHODOLOGY

Most of the large-scale cross-national studies on student achievement (e.g. TIMSS, PISA) as well as teacher preparation (e.g. TEDS-M) focused on complex data collection and employ, primarily, quantitative methods for data analysis. However, “to fully understand how achievement is contextualized in a given nation requires not only sets of complex data but also a range of analytical methods” (LeTendre, 2002). Therefore, the proposed study employed mixed methods sequential nested design (Tashakkori & Teddlie, 2003) and consisted of two stages: 1) quantitative stage was used for measuring teacher content knowledge; 2) qualitative stage was applied to analyze teacher responses on a set of open-ended questions in a topic-specific context - the division of fractions. For the first stage, quantitative data were collected and analyzed to further zoom into the qualitative analysis (the second stage of the study) of unpacking shared approaches as well as to address contested areas in teachers’ topic-specific content knowledge in the U.S. and Russia. The triangulation between the results of the two stages is further discussed in the Conclusion section.

In this section, we will describe the study participants, the instrument as well as data collection and data analysis procedures by two of its major stages: Stage 1 (using quantitative method) and Stage 2 (employing primarily qualitative method).

STAGE 1: QUANTITATIVE STUDY

PARTICIPANTS

The sample of the quantitative study for Stage 1 consisted of lower secondary mathematics teachers from the U.S. (grades 6–9, N=102) and Russia (grades 5–9, N=97). The U.S. teacher-participants were selected from urban public middle schools in the Southwestern part of the country. Teacher sample demographic information was self-reported by participating teachers. In terms of gender distribution, 55% of teacher participants were females and 45% – males. Most of the U.S. participants (64%) had 1–5 years of teaching experience. Additionally, 62% of the teacher sample received their teaching certificate through traditional teacher preparation programs and 38% of participating teachers were certified through alternative programs. The Russian teacher-participants were selected from urban public secondary schools in the Volga region. Russian participating teachers had attained a secondary mathematics teacher preparation Specialist's degree[1], which allowed them to teach in secondary schools (grades 5–11). The majority of participating teachers were females (89%). The sample was composed of 78% of teachers who have more than 10 years of teaching experience.

INSTRUMENT

The instrument used in this study was the Teacher Content Knowledge Survey which was developed using TIMSS framework (Mullis et al. 2016). It was designed to assess teacher content knowledge of Number, Algebra, Geometry, Data and Chance based on the three cognitive domains: Knowing, Applying, and Reasoning.

The instrument was developed by a group of interdisciplinary faculty with expertise in the following domains: mathematics, mathematics education, statistics, and statistics education, representing various institutions (university, community college, and local schools). Main steps in the item development process were the selection of items for the survey, the classification of items by cognitive domain, and modifying an item in other cognitive domains. To do so, we created a list of descriptors for each cognitive domain. The list for the Knowing domain included, but was not limited, to the following descriptors: recognize basic terminology and notation, recall facts, state definitions, name properties and rules, do computations, make observations, conduct measurements, simplify and evaluate numerical expressions. One may consider the following problem as an example of the Knowing domain question: what is the rule for fraction multiplication.

The list for the Applying domain consisted of the following descriptors: perform procedure (with or without connections), select and use appropriate representation, translate between representations, transform within the same representation, transfer knowledge to a new situation, connect two or more concepts, explain and justify solutions, communicate mathematical ideas, explain findings and results from analysis of data. Considering the same context of fraction multiplication, examples of the

[1] In Russia, the secondary school consists of lower and upper levels: the lower secondary school includes grades from 5 to 9, and grades 10–11 are part of the upper secondary school.

applying domain questions might be the following: a) multiply two given fractions (procedure without connection); and/or *b*) make up a story for the given fraction multiplication (procedure with connections).

And, the list for the Reasoning domain included the following descriptors: prove statements and theorems, solve non-routine problems, generalize patterns, formulate mathematical problems, generate mathematical statements, derive mathematical formulas, make predictions and hypothesize, design mathematical models, extrapolate findings from data analysis, test conjectures, to name a few. With regard to the same multiplication of fractions context, an example of the task at the reasoning domain level might be the following: is the following statement $\frac{a}{b} \times \frac{c}{d} = \frac{ad}{bc}$ *(a, b, c,* and *d* are positive integers) ever true?

In order to establish content validity, the specification table was constructed to guide the process of the instrument development. The table included major content topics and objectives for teachers closely aligned with corresponding objectives in lower secondary content standards. Aside from the specification table, the item analysis table was used to further ensure construct validity. The item analysis table included samples of competencies and items from the TCKS mapped and aligned with competencies from the lower secondary mathematics standards for students. A pool of 216 items was developed using this approach. The instrument was piloted with a sample of in-service lower secondary mathematics teachers enrolled in a graduate mathematics education course. After the pilot, the TCKS items were revised by a group of experts in mathematics, statistics, and mathematics education to finalize the instrument. A sample of 33 items was selected from the initial pool to be used in the study as the TKCS. The alpha coefficient technique (Cronbach, 1951) was utilized to evaluate the reliability of the TCKS. The obtained value of the coefficient at *α=0.839* suggests that the items comprising the survey are internally consistent (Author, 2011). As Hill, Ball, and Shilling (2008) claim, in education “reliabilities of *.70* or above are considered adequate for instruments intended to answer research and evaluation questions” (p. 386).

The final version of the TCKS survey consisted of 33 multiple-choice items addressing main topics of lower secondary mathematics curriculum: Number (9 items), Algebra (9 items), Geometry (9 items), Data and Chance (6 items) as well as different cognitive types of content knowledge: Knowing (10 items), Applying (13 items), and Reasoning (10 items).

INSTRUMENT TRANSLATION AND ADAPTATION

Initially, the TCKS instrument was developed, field tested and validated in the USA (Author, 2011). Considering that teaching is a cultural activity (Stigler & Hiebert, 1998), one should be sensitive to issues related to the adaptation of an instrument in different settings. Scholars (Andrews, 2011; Pepin, 2011) documented variations across countries in ways curriculum and content are structured, procedures and concepts are introduced, assignments of homework as well as individual and group work in the classroom are distributed, the blackboard is used during instruction, etc. Scholars apply different methods to validate and adapt an instrument in a new setting. Delaney et al. (2012) employed the teacher interviews to explore the consistency of teacher thinking and answer choices made using analysis of video recordings of lessons in order to examine the relationship between the teachers' scores and teaching practice. Moreover, the validity of an instrument heavily depends on the translation quality and linguistic equivalence (Pena, 2007). Therefore, we employed multi-level translation procedure using expertise of the Russian- speaking members of the research team to ensure linguistic equivalence of the adapted TCKS items with two rounds of independent translations followed by the round of reconciliation.

DATA COLLECTION

The measurement of teachers' knowledge was conducted using the TCKS instrument. Each teacher from participating countries was given 90 min to complete the survey. Along with teachers' scores on the TCKS, teachers' demographic information such as gender and ethnicity, years of teaching experiences, as well as other proxies for teacher content knowledge (i.e., mathematics coursework) were also collected.

DATA ANALYSIS

Taking into account the ordinal (ranked) nature of the quantitative data for content and cognitive domains of teacher knowledge (e.g., frequency counts) collected in the quantitative stage of the study, we used a non-parametric technique. This statistic was selected to measure the variance between independent groups of the same (not normal) distribution with arbitrary sample sizes of each group. In order to compare two or more groups (e.g., the U.S. and Russian teachers) on a response variable that is categorical in nature, it is suggested to apply the independent-samples Chi-square test (Huck, 2004, p. 463). This statistic detects group differences using frequency data. We also applied the Chi-square statistic to compare the responses of two independent groups of teachers to questions on the division of fractions during the second stage of the study.

STUDY 2: QUALITATIVE STUDY

The need for the qualitative stage is informed by the complexities in assessing teacher knowledge (Schoenfeld, 2007). One of the key issues is related to limitations of the multiple-choice format in test construction and assessment of teacher knowledge (p. 201). Responding to this limitation, we designed the qualitative stage to provide a closer examination of the U.S and Russian teachers' knowledge and understanding in the topic-specific context.

We selected the interpretive cross-case study design to examine the U.S. and Russian teachers' topic-specific knowledge of one of the important themes in lower secondary mathematics curriculum in both countries – the division of fractions. Merriam (1998) classified case studies with regard to its' overall intent as descriptive, interpretive, and evaluative. According to Merriam (1998), a descriptive case study presents "a detailed account of the phenomenon under study" (p. 38), an evaluative case study aims at "description, explanation, and judgment" (p. 39), and, finally, an interpretive case study focuses on "analyzing, interpreting, or theorizing about the phenomenon" (p. 38). Following the interpretive case study design, 16 teachers (eight from each country) were selected for the study after the completion of the TCKS.

Aside from taking TCKS in Stage 1, selected teachers were interviewed in Stage 2 on the topic of fraction division using questions addressing their content and pedagogical content knowledge. The cross-case analysis of teachers' topic-specific knowledge was conducted using meaning coding and linguistic analysis techniques (Kvale & Brinkmann, 2009).

PARTICIPANTS

A non-probability purposive sampling technique was employed to select study participants. Purposive sampling required that selected the U.S. and Russian teachers represent different quartiles of the total scores on the TCKS measure. It was also required that selected teachers teach at similar school settings (e.g. urban public schools).

With regard to the first criterion, the initial sample from both countries was subdivided by quartiles using teachers' overall TCKS scores. The distribution of the U.S. and Russian teachers' TCKS scores by quartiles are presented in Table 1.

Table 1. Distribution of the U.S. and Russian teachers' total TCKS scores by quartiles

Quartile	The U.S. teachers (N=102)			Russian teachers (N=97)		
	Range	N	%	Range	N	%
Q1	4-15	30	29	13-18	28	29
Q2	16-19	22	22	19-20	23	23
Q3	20-24	31	30	21-23	28	29
Q4	25-30	19	19	24-27	18	19

Table 1 indicates that the distribution of teachers across quartiles was similar to a third of the teachers in both the U.S. and Russian samples located in quartiles 1 and 3. There were 22% of the U.S. and 23% of Russian teachers located in quartile 2 and 19% of the teachers in each country located in quartile 4. We selected two teachers from each quartile after applying the purposive sampling criteria. Hence, the total study sample consisted of N=16 teachers (eight teachers from each country) who met the requirements of the purposive sampling. Selected teachers pseudonyms along with their total scores on TCKS across corresponding quartiles are presented in Table 2.

Table 2. Selected USA and Russian teachers' total TCKS scores by quartiles

Quartile	US Teachers		Russian Teachers	
	Pseudonym	Score	Pseudonym	Score
Q1	Rich Mary	13 15	Lera Inna	13 16
Q2	Grace Mark	18 19	Zina Victor	18 20
Q3	Lori Kate	21 23	Kiril Gala	21 22
Q4	Ron Sara	26 28	Anna Igor	25 27

Both the U.S. and Russian participants have similar teaching assignments – lower secondary school mathematics with content addressing the following main objectives: Number, Algebra, Geometry, Data and Chance. All selected teachers teach at urban public schools.

DATA COLLECTION

At Stage 2, we used the following data source – structured teacher interviews on the topic of the division of fractions. Teachers were interviewed using the following five questions related to the topic:

1) When you teach fraction division, what are important terms, facts, procedures, concepts, and reasoning strategies your students should learn?
2) What is the fraction division rule?
3) Apply the rule to the following fraction division problem: $1\frac{3}{4} \div \frac{1}{2} =$
4) Construct a word problem for the given fraction division: $1\frac{3}{4} \div \frac{1}{2} =$.
5) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c,* and *d* are positive integers) ever true?

The first question aimed at teachers' pedagogical content knowledge and focused on teachers' understanding of learning objectives for the topic of fraction division. The subset of questions 2)–5) assessed teachers' understanding of topic-specific content across the cognitive domains of knowing, applying, and reasoning.

DATA ANALYSIS

During the qualitative stage, teacher interviews were audio recorded and transcribed. In order to analyze qualitative data, we conducted meaning coding and linguistic analysis of teacher narratives as a primary method of analysis (Kvale & Brinkmann, 2009). The linguistic analysis technique unpacks "the characteristic uses of language, … the use of grammar and linguistic forms" (Kvale & Brinkmann, 2009, p. 219) by participating teachers within the specific topic of lower secondary mathematics. Additionally, the linguistic analysis was applied to check teacher use of mathematical terminology (questions 1–3). In order to "breaking down, examining, comparing, conceptualizing and categorizing data" (Strauss & Corbin, 1990, p. 61) we used data-driven meaning coding technique. This technique was applied to analyze teachers' responses on questions tapping into their understanding of meanings of the division of fractions (question 4) as well as their justification for solving the non-routine problem (question 5). To increase the credibility of the qualitative data analysis, the meaning coding and linguistic analysis were performed and cross-checked by two independent raters.

RESULTS

STAGE 1 FINDINGS

In this section, we first analyze teacher knowledge data by content domain, then we analyze teacher data by cognitive domain, and finally, we briefly discuss parallels between student and teacher performance within and between countries.

The results reported on teacher content knowledge show that the U.S. teachers' highest mean score was obtained on Number domain – 0.6230 and lowest on Geometry domain – 0.5142 (see Table 3).

Table 3. The U.S. teachers´ means scores by content domain

Content Domain	Mean	SE	SD	Conf. level (95%)
Number	0.6230	0.0203	0.2051	0.0403
Algebra	0.5632	0.0232	0.2347	0.0461
Geometry	0.5142	0.0254	0.2569	0.0505
Data and Chance	0.5931	0.0210	0.2118	0.0416

Russian teachers' highest mean score was obtained on Algebra domain – 0.7276 and lowest on Data and Chance domain – 0.3871 (see Table 4).

Table 4. Russian teachers´ means scores by content domain

Content Domain	Mean	SE	SD	Conf. Level (95%)
Number	0.6560	0.1066	0.3197	0.0239
Algebra	0.7276	0.0829	0.2487	0.0306
Geometry	0.5856	0.0727	0.2181	0.0455
Data and Chance	0.3871	0.1251	0.3064	0.0358

Moreover, we found that the U.S. teachers' highest mean score was obtained, as expected, on Knowing domain – 0.7343 and lowest on Reasoning domain – 0.4951 (see Table 5).

Table 5. The U.S. teachers´ means scores by cognitive domain

Cognitive Domain	Mean	SE	SD	Conf. level (95%)
Knowing	0.7343	0.0198	0.1977	0.0392
Applying	0.5053	0.0207	0.2071	0.0411
Reasoning	0.4951	0.0238	0.2381	0.0473

Russian teachers' highest mean score was obtained, as expected, on Knowing domain – 0.7598 and lowest, unexpectedly, on Applying domain – 0.5036 (see Table 6).

Table 6. Russian teachers´ means scores by cognitive domain

Cognitive Domain	Mean	SE	SD	Conf. level (95%)
Knowing	0.7598	0.0142	0.1352	0.0283
Applying	0.5036	0.0128	0.1214	0.0254
Reasoning	0.5928	0.0177	0.1683	0.0353

We used frequency counts of correct and incorrect responses to calculate Chi-square statistic in order to report differences between the U.S. and Russian teachers' performance on content and cognitive domains (Tables 7 and 8). With regard to content domain, we identified that there is no significant difference between Russian and the U.S. teachers' knowledge on Number domain (*$\chi2=2.1470$, $p>.05$*). However, there is a statistically significant difference between Russian and the U.S. teachers' knowledge on Data and Chance domain (in favor of the U.S. teachers; *$\chi2=50.914$, $p<.01$*) as well as Algebra and Geometry domains (in favor of Russian teachers correspondingly; *$\chi2=52.342$, $p<.01$* and *$\chi2=9.454$, $p<.01$*) (see Table 7).

Table 7. The U.S. and Russian teachers' knowledge by content domain usingfrequencies of correct/incorrect responses

Content Domain	Number Correct/ Incorrect	Algebra Correct/ Incorrect	Geometry Correct/ Incorrect	Data and Chance Correct/ Incorrect
Russia	573/300	636/237	512/361	225/357
USA	572/346	517/401	472/446	363/249
Chi-square	2.1470	52.342	9.454	50.914
p-value	0.1428	0	0.0021	0

This finding closely parallels the U.S. and Russian *students' performance* on TIMSS with regard to Data and Chance domain (in favor of the U.S. students) and Algebra domain (in favor of Russian students).

Additionally, the study reported that there is no significant difference between Russian and the U.S. teachers' knowledge on Knowing and Applying cognitive domains (*$\chi2=1.707$, $p>.05$ and $\chi2=0.008$, $p>.05$, correspondingly*) whereas there is a statistically significant difference on Reasoning domain (in favor of Russian teachers; $\chi2=19.117$, $p<.01$) (see Table 8).

Table 8. The U.S. and Russian teachers' knowledge by cognitive domain using frequencies of correct/incorrect responses

Cognitive Domain	Knowing Correct/ Incorrect	Applying Correct/ Incorrect	Reasoning Correct/ Incorrect
Russia	737/233	635/626	575/395
USA	749/271	670/656	505/515
Chi-square	1.707	0.008	19.117
p-value	0.1914	0.9287	0.000012

This finding also parallels the U.S. and Russian students' performance on TIMSS' cognitive domain (Mullis et al., 2016).

STAGE 2 FINDINGS

In this section, we present the U.S. and Russian teachers' responses to the questions on the division of fractions.

Teacher responses to Question 1

The Question 1 asked, "When you teach fraction division, what are the important terms, facts, procedures, concepts and reasoning strategies your students should learn?" Accordingly, teacher responses were coded using the following categories: 1) vocabulary, 2) knowing, 3) applying, and 4) reasoning. The frequency of teacher responses in each category with reported chi-square statistic[2] and p-values are presented in Table 9.

[2] In the 2x2 contingency case of the chi-square test, for expected frequencies less than 5 Yates' correction is employed.

Table 9. The frequency of the U.S. and Russian teachers' responses to Question 1 by categories

Category	The U.S. teachers	Russian teachers	χ^2
Vocabulary	33	38	*2.003*
Knowing	27	32	*3.471*
Applying	20	23	*0.893*
Reasoning	0	6	*6.667***

**p<.05, **p<.01*

Most frequently used category in response to Question 1 was "vocabulary" with the total amount of counts – 71: 33 counts in the U.S. teachers' responses and 38 counts in Russian teachers' responses with no significance observed between the groups ($\chi^2=2.003$, $p>.05$). Most frequently used terms emerged from teachers' responses are "division" (9 counts), "reciprocal" (11 counts), "denominator" (8 counts), "multiplication" (7 counts). Least frequently used terms are "dividend" (3 counts), divisor (3 counts), "quotient" (3 counts). With regard to categories "knowing" and "applying", we also didn't detect any significant differences between the groups: chi-square values $\chi^2=3.471$ and $\chi^2=0.893$ at $p>.05$ correspondingly. The only category where the significance was observed in the category of "reasoning" ($\chi^2=6.667$, $p<.01$). In the Discussion and Conclusion section of the paper, we will analyze these findings in more detail.

Teacher responses to Question 2

The second question asked teachers to respond to the following: what is the fraction division rule. In Table 10 we present the frequency of terms used by the U.S. and Russian teachers while explaining the rule for fraction division along with chi-square values for each reported term.

Table 10. The frequency of terms used by the U.S. and Russian teachers in response to Question 2

Terms used by teachers	The U.S. teachers	Russian teachers	χ^2
Flip	7	1	*6.250**
Reciprocal	7	8	*1.067*
Dividend	0	6	*6.667***
Divisor	0	6	*6.667***
First fraction	6	2	*2.250*
Second fraction	6	2	*2.250*
Quotient	0	1	*1.067*

*$p<.05$, **$p<.01$

All U.S. and Russian teachers correctly responded to this question. However, the way they described the rule deserves a separate discussion which we will provide in the Discussion and Conclusion section.

Teacher responses to Question 3

As expected, teachers' responses to the procedural question 3 (divide two given fractions) were the least insightful. Most of the teachers in both groups silently performed the division on a scratch paper that was provided to every participant. All participating teachers correctly solved the given fraction division task. Slight differences were observed in the representation of the answer. Whereas all eight U.S. teachers wrote the answer in mixed number form as 3,5 only two Russian teachers did the same. Five Russian teachers wrote the answer in decimal form 3.5 and one Russian teacher wrote the answer in both forms 3,5. One observation deserves mentioning and further discussion: one of the U.S. teachers illustrated the division by a pictorial model (see the Discussion and Conclusion section).

Teacher responses to Question 4

Question 4 tapped into teachers' understanding of meaning(s) of the division of fractions while asking them to construct a word problem for the given problem. There are several distinct meanings of the division of fractions discussed by scholars. For instance, Fischbein et al. (1985) and Simon (1993) identified two main meanings for the division of fraction: quotitive (measurement) and partitive (part-to-whole). At the same time, Greer (1992) proposed to consider the "rectangular area" model within the partitive meaning of the fraction division. Later Ma (1999) included the rectangular model in a separate category, which she called “product and factors”. Therefore, Ma claimed that there are three main models and corresponding meanings to represent the division of fractions: measurement, partitive, and product and factors (1999, p. 72).

We observed that question 4 was challenging to the U.S. teachers – only five teachers were able to construct a correct word problem compared to eight Russian teachers. An insightful observation was recorded in models used by teachers to construct a word problem which will be further discussed later in the Discussion and Conclusion section. In Table 11, we include frequencies of meanings/ models used by the teachers to construct word problems along with chi-square statistic and p-values.

Table 11. The frequency of meanings of fraction division used by the U.S. and Russian teachers in response to Question 4

Meanings of fraction division	The U.S. teachers	Russian teachers	χ^2
Part-to-whole (partitive)	0	2	*0.571*
Measurement (quotitive)	5	2	*2.286*
Rectangular area model (product and factors)	0	4	*5.333**
Incorrect	3	0	*3.692*

**p<.05, **p<.01*

Chi-square analysis showed statistically significant difference not only for the rectangular area model but also an overall difference in performance of the U.S. and Russian teachers on this particular task (*$\chi 2$ = 10.286, p<.05).*

Teacher responses to Question 5

Question 5 aimed at assessing teachers’ critical reasoning: is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c,* and *d* are positive integers) ever true? This question was challenging to both the U.S. and Russian teachers. Table 12 captures frequencies of solutions/ proofs proposed by teachers along with the chi-square values.

Table 12. The frequency of solutions proposed by the U.S. and Russian teachers in response to Question 4

Teacher responses	The U.S. teachers	Russian teachers	χ^2
Never true	5	4	*0.254*
True if a=b=c=d	1	1	*0*
True if c=d	1	3	*1.333*
No solution provided	1	0	*1.067*
Using numerical values to prove	4	0	*5.333**

$p<.05$, $**p<.01$

As depicted in Table 12, we were not able to observe any significant differences between groups in a number of correct responses to Question 5 (only one correct and one partially correct solution proposed by the U.S. teachers compared to three correct and one partially correct solutions provided by Russian teachers). However, an interesting observation was recorded with regard to a method of proof used by teachers which we will elaborate further on in the Discussion section.

DISCUSSION

In this section, we discuss the major results of the study, emphasize the triangulation of the results between the stages. Further, we share some insightful observations related to every question we used during the teacher interviews in Stage 2.

We will start with observations on teacher articulation of the learning objectives for the topic of fraction division (Question 1). Then we will discuss teacher use of mathematical vocabulary, facts and procedures (Questions 1–3). We will proceed to teacher understanding of meaning(s) of the division of fractions. Finally, we will address the observation of methods employed by teachers while responding to the reasoning Question 5.

Teacher articulation of the learning objectives for the division of fractions

Most insightful finding in teachers' responses to Question 1 was the fact that both the U.S. and Russian teachers quite similarly defined learning objectives for the division of fraction. Both groups clearly outlined the main vocabulary students should learn, facts and procedures students should master, and concepts students should understand. The revealing difference was observed in the teachers' response to the reasoning category. Despite the fact that Question 1 explicitly asked to articulate "what are important … reasoning strategies your students should learn?", none of the U.S. teachers responded to this part of the question compared to six Russian teachers who highlighted the importance of "developing logical reasoning" (4 teachers) as well as "checking for reasonableness" (2 teachers). This finding may suggest that the U.S. teachers do not see a "reasoning" potential in the topic of the division of fractions whereas their Russian counterparts emphasize the development of students' reasoning as one of the important learning objectives for the topic of fraction division.

Teacher use of mathematical vocabulary related to the division of fractions

As mentioned earlier, both the U.S. and Russian teachers emphasized the importance of developing students' mathematical vocabulary related to the topic of the division of fractions. Table 14 captures frequencies of terms used by teachers in both groups.

Between two groups of teachers, there were 13 terms recorded in response to the "vocabulary" category of the Question 1 as indicated in Table 7. We thought that several observations deserve a further discussion. First, most frequently used term among the U.S. teachers was "division" (6 frequency counts) whereas "reciprocal" (7 counts) and "multiplicative inverse" (6 counts) were the most frequently used terms by Russian teachers. This result may suggest that the U.S. teachers focused on the operation in general (e.g. division) whereas Russian teachers emphasized the operation specific to the division of fractions (e.g. reciprocal, multiplicative inverse). We also noticed that Russian teachers were using the terms reciprocal and multiplicative inverse interchangeably. It may suggest that Russian teachers use these synonyms with some level of distinction. Indeed, from a mathematical perspective, the term "reciprocal" has a specific meaning: the reciprocal of x is $1/x$. For instance, the reciprocal of 2 is 1/2 the same way as the reciprocal of 1/2 is 2. At the same time, the term "multiplicative inverse" is more general for the very reason that the term "inverse" means something that is opposite to something. For example, subtraction is an inverse operation to addition, the same way as multiplication is an inverse operation to division. Perhaps, Russian teachers explicitly used multiplicative inverse in the context of the division of fraction to distinguish it from additive inverse.

The second observation concerns the elements of the division operation. Even though the term division as the operation was most frequently used by the U.S. teachers, none of them reported elements of this operation in their responses. Opposite to this, three Russian teachers referred to the elements of the division operation (e.g. dividend, divisor, and quotient) as an important learning objective to reinforce while studying the division of fractions.

Table 14. The frequency of vocabulary terms used by the U.S. and Russian teachers in response to Question 1

	Vocabulary terms	The U.S. teachers	Russian teachers
1.	Parts of a whole	3	3
2.	Division	6	3
3.	Numerator	1	5
4.	Denominator	3	5
5.	Reciprocal	4	7
6.	Improper fraction	1	4
7.	Mixed number	1	4
8.	Multiplication	3	4
9.	Multiplicative inverse	2	6
10.	Factor/ Product	0	3
11.	Dividend	0	3
12.	Divisor	0	3
13.	Quotient	0	3

Accurate use of mathematical terms by Russian teachers was also evident in the response to Question 2. Even though all the U.S. and Russian teachers correctly responded to this question, the way they described the rule deserved a close examination. First, we observed that despite low frequency in using terms “reciprocal” and “multiplicative inverse” in response to Question 1, the U.S. teachers recalled the term “reciprocal” more frequently (7 counts) in response to Question 2. Next observation is concerned with the use of accurate mathematical terminology: “dividend” vs. “first fraction” and “divisor” vs. “second fraction” which was statistically significant in both cases as depicted in Table 4. Third, our observation revealed a strong tendency on the part of the U.S. teachers to use the term “flip” as a sub-language for reciprocal/ multiplicative inverse with a reported chi-square value of $\chi^2=6.250$ at the significance level $p<.05$. Last but not least, we were pleased to receive the pictorial representation of fraction division performed by Kate – the U.S. teacher – in response to Question 3 using the measurement model of fraction division as depicted in Figure 1.

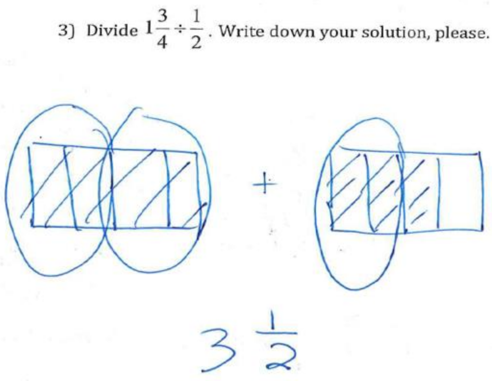

Fig. 1. Kate’s drawing to represent the given fraction division problem

It is interesting to note that, first, Kate decided to represent $1\frac{3}{4}$ as $1+\frac{3}{4}$ and, then, skillfully show how many times $\frac{1}{2}$ will go into $1\frac{3}{4}$ to obtain the quotient value of $3\frac{1}{2}$ as illustrated by the circled shaded blocks in the picture.

Teacher understanding of meaning(s) of the division of fractions

Following on the previous discussion on Kate’s visual representation of the measurement model of fraction division, we found that the measurement model was the most popular model (5 frequency counts as presented in table 5) and the only one model used by the U.S. teachers in response to Question 4 asking to construct a word problem for the given problem $1\frac{3}{4} \div \frac{1}{2} =$. In contrast, Russian teachers applied all three models for the fraction division meaning proposed by Ma (1999) with the product and factors/rectangular area model being the statistically significant one with a chi-square value of $\chi^2=5.333$ at $p<.05$. Examples of word problems constructed by teachers using different models of the division of fractions are presented below:

- quotitive/measurement model: “*Students have $1\frac{3}{4}$ yards of ribbon. They plan on using the ribbon to make bows for kites. If each bow requires $\frac{1}{2}$ yard of ribbon, how many bows will they be able to make*” (the word problem constructed by the U.S. teacher Mary);

- partitive/part-t-whole model: “*Farmers collected* $1\frac{3}{4}$ *tons of grain from the field of* $\frac{1}{2}$ *hectares. How much grain they could collect from 1 hectare*” (the word problem constructed by the Russian teacher Kiril);
- rectangular area/product-and-factors model: “*The area of a rectangle is* $1\frac{3}{4}$ *cm*^{2}*. Find its length if the width is equal to* $\frac{1}{2}$ *cm*” (the word problem constructed by the Russian teacher Zina).

Teacher reasoning in the fraction division context

Analysis of teacher narratives to question 5 did not show significant differences between groups in a number of correct responses. Whereas the U.S. teachers proposed only one correct (c=d) and one partially correct solution (*a=b=c=d*), their Russian counterparts provided three correct and one partially correct solutions. An example of the solution, rated by experts as the correct one, provided by Sara is presented in Figure 2.

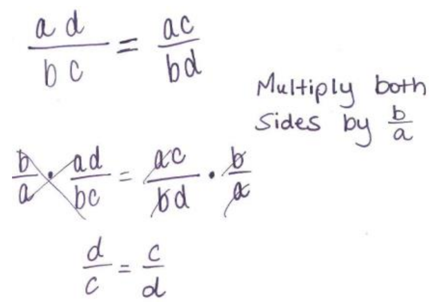

Fig. 2. Sara’s proof to Question 5

A statistically significant difference ($\chi^2=5.333$, $p<.05$) was reported with regard to a method of proof used by teachers. None of the Russian teachers attempted to prove the statement numerically compared to four U.S. teachers who tried to plug in different numbers to check if the statement works. An example of the numerical attempt to solve Question 5 is illustrated in Figure 3.

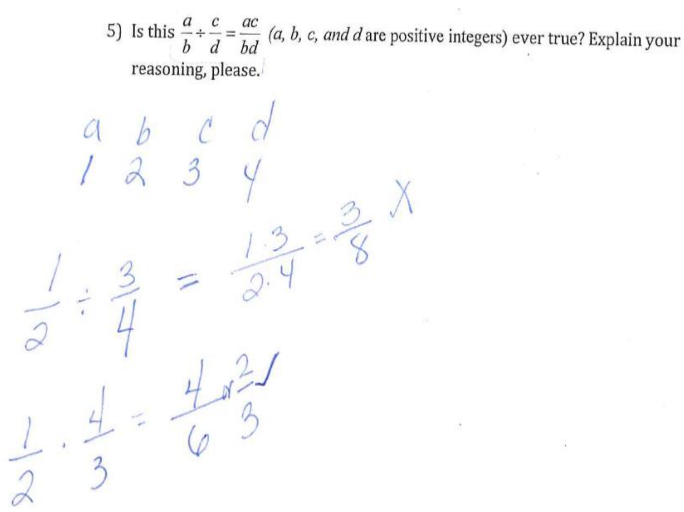

Fig. 3. Grace’s numerical approach to solve Question 5

Also, there was one episode of not offering any solution to Question 5 among the U.S. teachers which was not a case among Russian teachers.

CONCLUSION

In this section, we will provide concluding remarks on the main findings of the study with emphasis on its implication, limitation, triangulation, and significance.

As reported in Results section, this study confirms the differences between Russian and the U.S. lower secondary in-service teachers’ knowledge in the content domain similar to the findings reported by the TEDS-M study that was focused on pre-service teachers (Tatto & Senk, 2011). At the same time, this study expands the examination of in-service teachers’ knowledge to the cognitive domain.

IMPLICATION

Teacher preparation could be considered as the main factor contributing to the differences between Russian and the U.S. teachers’ knowledge. Overall, there is a tangible difference in secondary teacher preparation curriculum between the two countries: on average, Russia offers about 240 credit hours in teacher preparation programs compare to 120 credits in the USA. Furthermore, cross-national curriculum analysis shows that Russian teachers have more extensive content preparation compare to their American counterparts. A number of contact hours for mathematical content knowledge, as well as pedagogical content knowledge and specialized mathematics knowledge offered at selected teacher preparation programs located in the regions/ states where the study was conducted, are presented in Table 13.

Table 13. Contact hours in Mathematics related disciplines in selected teacher education programs in Russia and the United States

Country	Mathematics Content Knowledge (Academic Mathematics)	Pedagogical Content Knowledge (Mathematics Pedagogy)	Specialized Mathematics Knowledge (School Mathematics)
Russia	1857	278	380
United States	442	72	87

Numbers depicted in the table are compatible with the findings of the TEDS-M study (Tatto & Senk, 2011).

Close examination of the secondary teacher preparation curriculum in Russia shows more emphasis placed on an analytic and algebraic component of mathematics and less emphasis on statistic and probability component compare to the U.S. curriculum. Moreover, item analysis of standardized tests for the lower secondary schools in USA and Russia revealed the difference in selection and composition of algebra problems as well as problems related to data and chance in the test: while in Russia more emphasis is placed on algebraic problems and less emphasis on data and chance problems, in the USA – the emphasis is equally distributed among algebraic problems and data and chance problems. We observed another noticeable difference in the role of formal proof in the academic mathematics component of the teacher preparation program which could explain the difference in the reasoning domain of the teacher knowledge: traditionally, Russian curriculum places a heavy emphasis on logic and formal proof across the mathematics coursework including school mathematics whereas the U.S. curriculum uses proof in selected mathematics courses primarily in academic mathematics coursework.

LIMITATION

We are cognizant of the limitations concerning the convenient sampling technique that influences the generalizability of the study results. Moreover, there is no cluster matching between teachers participating in the study and students tested in

TIMSS. However, the study’s Stage 1 main results suggest that student performance on international tests could be explained by teacher knowledge. Considering the qualitative nature of the research design employed in Stage 2, we are cognizant of the limitations of the study sampling (e.g. purposing sampling) and, congruently, we are sensitive do not overgeneralize its results as well.

TRIANGULATION

Synthesizing the main findings of the study through the triangulation lens, we report that the topic-specific level of analysis at Stage 2 helped us to unpack hidden insights in terms of differences and similarities in teacher content knowledge among participants in the U.S. and Russia obtained at Stage 1. The granualized methodology used in the study to unpack and analyze teacher topic-specific knowledge could be considered as a potential contribution to the field of cross-national studies on teacher knowledge.

SIGNIFICANCE

Overall, the study findings revealed that there are similarities and differences in teachers’ content knowledge as well as its cognitive types. The results are reflected in meanings expressed and the language used by teachers while responding to topic-specific questions on the division of fractions. The results of the study suggest that in the cross-national context teachers’ knowledge could vary depending on curricular as well as socio-cultural priorities placed on teaching and learning of mathematics.

The study also presents opportunities for comparing, sharing, and learning about issues in cross-national context in the U.S. and Russian teacher education, training, and development. Moreover, the reported cross-national study on teacher knowledge may inform the field on priorities placed on lower secondary mathematics teachers’ knowledge in the USA and Russia by content and cognitive domains.

The study main findings contribute to a body of literature in the field of cross-national research on teacher knowledge with a narrow focus on topic-specific knowledge. It suggests close comparison and learning about issues related to teacher knowledge in the U.S. and Russia with a potential focus on re-examining practices in teacher preparation and professional development.

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